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Superconductivity in Restricted Chromo-Dynamics (RCD) in SU(2) and SU(3) Gauge Theories

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Abstract Characterizing the dyonically condensed vacuum by the presence of two massive modes (one determining how fast the perturbative vacuum around a colour source reaches the condensation and the other giving the penetration length of colored flux) in SU(2) theory, it has been shown that due to the dynamical breaking of magnetic symmetry the vacuum of RCD acquires the properties similar to those of relativistic superconductor. Analysing the behaviour of dyons around RCD string, the solutions of classical field equations have been obtained and it has been shown that magnetic constituent of dyonic current is zero at centre of the string and also at the points far away from the string. Extending RCD in the realistic color gauge group SU(3), it has been shown that the resulting Lagrangian leads to dyonic condensation, color confinement and the superconductivity with the presence of two scalar modes and two vector modes.

Keywords Restricted chromo-dynamics · Penetration length · Dyons · Confinement · Condensation · Meissner effect

1 Introduction

In the process of current understanding of superconductivity at high T_c, one conceives the notion of its hopeful analogy with quantum chromo-dynamics (QCD). The essential clues for gauge symmetry breaking emerged from the crucial theoretical frame work of BCS theory [1] of superconductivity. Other silent features of superconductivity viz. the Meissner effect and the flux quantization provided the vivid models for actual confinement mechanism. In this connection Nambu [2, 3] and others [4–6] suggested that the colour confinement could occur in QCD in a way similar to magnetic flux confinement in superconductors. Mandelstam [7–9] elaborated it by expounding that the colour confinement properties may result from the condensation of magnetic monopoles in QCD vacuum. In a series of papers Ezawa and Iwazaki [10–13] made an attempt to analyse a mechanism of quark confinement

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by demonstrating that the Yang-Mills vacuum is a magnetic superconductor and such a superconducting state is considered to be a condensed state of monopoles or magnetic vortices. The condensation of monopoles incorporates the state of magnetic superconductivity [14] and the notion of chromo-magnetic superconductor [15] where the Meissner effect confining magnetic field in ordinary superconductivity would be replaced by dual Meissner effect which would confine the colour electric field. Its leads to a correspondence between quantum chromo-dynamic situation and chromo-magnetic superconductor where the Abelian electric field is squeezed by solenoidal monopole current [16, 17] and the colour confinement takes place due to the dual Meissner effect caused by monopole condensation.

Using this idea of confinement of electric flux due to condensation of magnetic monopoles, a dual gauge theory called restricted chromo dynamics (RCD) has been constructed out of QCD in SU(2) theory [18–21]. This dual gauge theory incorporates a dynamical dyonic condensation [22, 23] and exhibits the desired dual dynamics that guarantees the confinement of dyonic quark through generalized Meissner effect. This RCD has been extracted from QCD by imposing an additional internal symmetry named magnetic symmetry [18, 24] which reduces the dynamical degrees of freedom. Attempts have been made [25] to establish an analogy between superconductivity and the dynamical breaking of magnetic symmetry, which incorporates the confinement phase in RCD vacuum.

In the present paper the formulation of RCD has been extended in the light of the concept of chromo-dyonic superconductor and it has been shown that in the confinement phase the dyonic condensations of vacuum gives rise to the complex screening current which confines both the chromo-electric and chromo-magnetic fluxes through the mechanism of generalized Meissner effect (the usual one and its dual). Characterizing the dyonically condensed vacuum by the presence of two massive modes (one determining how fast the perturbative vacuum around a colour source reaches the condensation and the other giving the penetration length of the coloured flux), it has been shown that due to the dynamical breaking of magnetic symmetry the vacuum acquires the properties similar to those of relativistic super-conductor where the quantum fields generate non-zero expectation values and induce screening currents. It has also been shown that the generalized charge space parameter associated with dyons has the remarkable ability to squeeze the colour fluxes and to improve the confining properties of RCD vacuum. Analyzing the behaviour of dyons around the RCD string with a quark and an anti- quark at its ends, the solutions of classical field equations have been obtained removing the mistakes in the recent relations of Chernodub et al. [26] and it has been shown that the magnetic constituent of the dyonic current is zero at centre of the string and also at points far away from the string. Extending RCD in the realistic color gauge group SU(3) by using two internal Killing vectors a λ_3-like octed and a λ_8-like octed, the RCD Lagrangian of SU(3) theory has been obtained in magnetic gauge and it has been shown to lead to dyonic condensation, color confinement and the resulting superconductivity in SU(3) theory with the presence of two scalar modes and two vector modes.

2 Magnetic Symmetry and Restricted Chromodynamics (RCD) in SU(2) Gauge Theory

Mathematical foundation of RCD [18, 21] is based on the fact that a non-Abelian gauge theory permits some additional internal symmetry i.e. the magnetic symmetry. Let us briefly review the RCD in the $(4+n)$ dimensional metric manifold P (four-dimensional space-time manifold M and n-dimensional internal group G) with metric g_{AB} $(A, B = 1, 2, \ldots 4+n)$, where the gauge symmetry can be viewed as n-dimensional isometry [27, 28] which allows us to view P as a principal fibre bundle $P(M,G)$ with $M=P/G$ as the base manifold and G as the structure group. Keeping is view the fact [21] that the restricted theory RCD may be extracted from full QCD by imposing an extra internal symmetry, let us now restrict the dynamical degrees of freedom of the theory (keeping full gauge degrees of freedom intact) by imposing an extra magnetic symmetry which ultimately forces the generalized non-Abelian gauge potential $\vec{V}_\mu$ to satisfy a strong constraint given by

$$D_\mu \hat{m} = \partial_\mu \hat{m} + i|q|\vec{V}_\mu \times \hat{m} = 0 \tag{2.1}$$

where D_μ is covariant derivative for the gauge group, $\mu = 0,1,2,3$, $q = (e - ig)$ is the generalized dyonic charge with e and g as electric and magnetic constituents, and the generalized four-potential $\vec{V}_\mu$ is given as

$$\vec{V}_\mu = \vec{A}_\mu - i\vec{B}_\mu \tag{2.2}$$

where A_μ and B_μ are electric and magnetic four-potentials respectively. The cross product in (2.1) is taken in internal group space and $\hat{m}$ characterizes the additional Killing symmetry-(magnetic symmetry) which commutes with the gauge symmetry itself and is normalized to unity i.e.

$$\hat{m}^2 = 1 \tag{2.3}$$

It constitutes an adjoint representation of G, whose Little group is assumed to be Cartan sub-group [18] at each space-time point. Mathematically, this means that a connection on $P(M,G)$ admits a left isometry of H, which formally forms a subgroup of G but commutes with G (the right isometry). This magnetic symmetry restricts the connection (i.e. the space for potential) to those whose holonomy bundle becomes a reduced bundle $P(M,H)$.

Choosing $G = \mathrm{SU}(2)$ and $H = \mathrm{U}(1)$, the gauge covariant condition (2.1) gives the following form of the generalized restricted potential,

$$\vec{V}_\mu = -iV_\mu^* \hat{m} + (i/|q|)\hat{m} \times \partial_\mu \hat{m} \tag{2.4}$$

such that $m \cdot \vec{V}_\mu = -iV_\mu^*$ is the unrestricted Abelian component of the restricted potential $\vec{V}_\mu$ while the remaining part is completely determined by magnetic symmetry.

The unrestricted part of the gauge potential describes the dyonic flux of color isocharges and the restricted part describes the flux of topological charges of the symmetry group G. The imposed magnetic symmetry, revealing the global topological structure of gauge symmetry, enables us to conceive the gauge theory of non-trivial fibre bundle $P(M,H)$ with only those fields which are defined on global sections where color direction would be chosen by selecting color electric potential of Cartan's sub-group which helps to circumvent the disturbing Schlieder's theorem [29] in defining a meaningful color charge in non-Abelian gauge theory.

The generalized field strength of the gauge field of RCD that describes non-Abelian dyons may be obtained as follows;

$$\begin{aligned} \vec{\cancel{G}}_{\mu\upsilon} &= \vec{G}_{\mu\upsilon} + (i/|q|)[\vec{V}_\mu \times \vec{V}_\upsilon] \\ &= (-iF_{\mu\upsilon} + H_{\mu\upsilon})\hat{m} \end{aligned} \tag{2.5}$$

where

$$\vec{G}_{\mu\upsilon} = \vec{V}_{\upsilon,\mu} - \vec{V}_{\mu,\upsilon},$$
$$F_{\mu\upsilon} = V^*_{\upsilon,\mu} - V^*_{\mu,\upsilon}, \tag{2.6}$$

and

$$H_{\mu\upsilon} = (i/|q|)\hat{m} \cdot [\partial_\mu \hat{m} \times \partial_\upsilon \hat{m}]$$

Identifying $F_{\mu\upsilon}$ and $H_{\mu\upsilon}$ as the generalized electric and magnetic field strengths respectively, the striking duality between the generalized electric and magnetic fields is obviously manifested in the theory. These field strengths satisfy the following dual symmetric field equations in magnetic gauge

$$F_{\mu\upsilon,\upsilon} = j_\mu \qquad \text{and} \qquad H_{\mu\upsilon,\upsilon} = -k_\mu \tag{2.6a}$$

where j_μ and k_μ are respectively the electric and magnetic four-current densities constituting the generalized dyonic four-current density

$$J_\mu = j_\mu - ik_\mu \tag{2.6b}$$

In order to demonstrate the topological structure, let us introduce magnetic gauge by aligning $\hat{m}$ along a space-time independent direction (say $\hat{\varepsilon}_3$ in isospin space) by imposing a gauge transformation U such that

$$\hat{m} \xrightarrow{\mathrm{U}} \hat{\varepsilon}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.7}$$

and the potential and field strength transform as

$$\vec{V}_\mu \to \vec{V}^{\mathbf{z}}_\mu = (-iV^*_\mu + W_\mu)\hat{\varepsilon}_3$$

and

$$\vec{G}_{\mu\upsilon} = \vec{G}^{\mathbf{z}}_{\mu\upsilon} = (-iF_{\mu\upsilon} + H_{\mu\upsilon})\hat{\varepsilon}_3 \tag{2.8}$$

with

$$H_{\mu\upsilon} = W_{\upsilon,\mu} - W_{\mu,\upsilon} \tag{2.9}$$

where W_μ may be identified as the potential of topological dyons in magnetic symmetry which is entirely fixed by $\hat{m}$ upto Abelian gauge degrees of freedom. Thus in the magnetic gauge, the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_μ by removing all non-essential gauge degrees of freedom and hence the topological structure of the theory may be brought into dynamics explicitly. It assures a non-trivial dual structure of the theory of dyons in magnetic gauge where dyons appear as point like Abelian ones. In this theory the gauge fields are expressible in terms of purely time like non-singular physical potentials V^*_μ and W_μ. Following Mandelstam [7] and t' Hooft [4], let us introduce a complex scalar field ϕ (Higg's field) to eliminate the point like behaviour and to incorporate the extended structure of dyons. Then in the absence of quarks or any colored object, the RCD Lagrangian in magnetic gauge may be written as

$$L = (1/4)H_{\mu\upsilon}H^{\mu\upsilon} + (1/2)|D_\mu\phi|^2 - V(\phi^*\phi) \tag{2.10}$$

where $D_\mu\phi = (\partial_\mu + i|q|W_\mu)\phi$ and $V(\phi^*\phi)$ is the effective potential introduced to induce the dynamical breakdown of the magnetic symmetry.

3 Dyonic Condensation and Super-Conductivity in Restricted Chromodynamics in SU(2) Gauge Theories

The Lagrangian (2.10) of RCD in magnetic gauge in the absence of quark or any colored object, looks like Ginsburg-Landau Lagrangian for the theory of superconductivity if we identify the dyonic field as an order parameter and the generalized potential W_μ as the electric potential. The dynamical breaking of the magnetic symmetry, due to the effective potential $V(\phi * \phi)$, induces the dyonic condensation of the vacuum. This gives rise to the dyonic super current, the real part of which (electric constituent) screens the electric flux which confines the magnetic color charge (through usual Meissner effect) and the imaginary part (i.e. magnetic constituent) of this super-current screens the magnetic flux that confines the electric color iso- charges (due to dual Meissner effect).

Lagrangian (2.10) has been obtained from the standard SU(2) Lagrangian and hence the desired dynamical breaking of magnetic symmetry is obtained by fixing the following form of the effective potential;

$$V(\phi^*\phi) = -\eta(|\phi|^2 - v^2)^2 \tag{3.1}$$

where η is compiling constant of Higgs field and v is its vacuum expectation value i.e.

$$\upsilon = \langle\phi\rangle_0 \tag{3.2}$$

In Prasad-Sommerfeld limit [30],

$$V(\phi) = 0$$

but $\upsilon \neq 0$.

In this limit, the dyons have lowest possible energy for given electric and magnetic charges e and g respectively. In this Abelian Higgs model of RCD in magnetic symmetry, W_μ defined by (2.9) is the dual gauge field with the mass of dual gauge boson given by

$$M_D = |q|v \tag{3.3}$$

and ϕ is the dyonic field with charge q and mass

$$M_\phi = \sqrt{(8\eta)}v \tag{3.4}$$

In the confinement phase of RCD the dyons are condensed under the condition (3.2). With these two mass scales the coherence length ε and the penetration length λ are given by

$$\varepsilon = 1/M_\phi = 1/[\sqrt{(8\eta)}v]$$

and (3.5)

$$\lambda = 1/M_D = 1/(|q|v)$$

The region in phase diagram space, where $\varepsilon = \lambda$, constitutes the border between type-I and type-II super-conductors. The super-conductivity provides vivid model for the actual confinement mechanism where the color confinement is due to the generalized Meissner effect caused by dyonic condensation.

The Lagrangian of (2.10), with effective potential given by (3.1), yields the following field equations;

$$\partial_\upsilon H^{\mu\upsilon} - i|q|[\phi^* D^\mu \phi] = 0 \tag{3.6}$$

and

$$D_\mu^2 \phi - 4\eta[|\phi|^2 - v^2]\phi = 0 \tag{3.7}$$

Equation (3.6) may also be written as

$$\Box W_\mu - \partial^\upsilon \partial_\mu W_\upsilon = k_\mu \tag{3.8}$$

where k_μ, the magnetic constituent of generalized dyonic current, is given as

$$k_\mu = |q|\mathrm{Im}[\phi^* D^\mu \phi] = |q||\phi|^2[\partial_\mu \arg\phi + |q|W_\mu] \tag{3.9}$$

In the Lorentz gauge, (3.8) reduces to

$$\Box W_\mu = i|q|\phi^*\phi[(\partial_\mu \phi)/\phi + i|q|W_\mu]$$

which furtherer reduces to the following form for the small variation in ϕ;

$$\Box W_\mu + |q|^2|\phi|^2 W_\mu = 0 \tag{3.10}$$

which is a massive vector type equation where the equivalent mass of the vector particle state (condensed mode) may be identified as

$$M^2 = |q|^2|\phi|^2$$

with its vacuum expectation value

$$\langle M\rangle = |q|v = M_D = 1/\lambda \tag{3.11}$$

where λ is penetration length defined by (3.5).

In the confinement phase dyons are condensed and

$$|\langle\phi\rangle| = v$$

Comparing the penetration length (i.e. screening length) λ of (3.11) with that of relativistic super conductor model i.e.

$$M_S = \sqrt{2}e|\phi| = \sqrt{2}ev = 1/\lambda_S \tag{3.12}$$

where e is the electric charge of dyons, we get

$$M/M_S = \lambda_S/\lambda = |q|/(e\sqrt{2}) = (1/\sqrt{2})[(e^2 + g^2)^{1/2}]/e \tag{3.13}$$

In the representation of generalized charge of dyon in a two dimensional complex space [31], we have

$$g/e = -\tan\theta \tag{3.14}$$

where θ is rotation parameter of the generalized charge space.

Then (3.13) gives

$$\lambda/\lambda_S = \sqrt{2}\cos\theta \tag{3.15}$$

showing that for the rotation parameter $\theta \leq \pi/4$, we have

$$\lambda \geq \lambda_S \qquad \text{and} \qquad M \leq M_S \tag{3.16}$$

On the other hand, for larger rotation in generalized charge space with $\theta > \pi/4$, we have

$$\lambda < \lambda_S \qquad \text{and} \quad M > M_S \tag{3.17}$$

Thus the optimum RCD generalized charge orientation is governed by rotation parameter value $\theta = \pi/4$.

Dyonically condensed vacuum is characterized by the presence of two massive modes. The mass of the scalar mode, M_ϕ given by (3.5) determines how fast the perturbative vacuum around a colored source reaches condensation and the mass M_D of vector mode determines the penetration length of the colored flux. The masses of these generalized dyonic glueballs may be estimated [20, 32] by evaluating string tension of the classical string solutions of quark pairs. For this let us examine the behaviour of dyons around the RCD string. The classical field equations (3.7) and (3.8) contain a solution corresponding to the RCD string with a quark and an anti-quark at its ends. Let us consider the static solution, parallel to the third direction of reference frame, as

$$\phi(\rho) = v f(\rho) e^{i\Psi} \tag{3.18}$$

and

$$W_1 = \hat{x}_2 h(\rho)/(|q|\rho^2), \quad W_2 = -\hat{x}_1/(|q|\rho^2) h(\rho), \quad W_3 = 0, \quad W_4 = 0 \tag{3.19}$$

where

$$\rho = (x_1^2 + x_2^2)^{1/2}$$

is the transverse distance to the string;

$$\Psi = \arg(x_1 + i x_2) \tag{3.20}$$

and

$$\begin{aligned} &\lim_{\rho\to 0} f(\rho) = \lim_{\rho\to 0} h(\rho) = 0, \\ &\lim_{\rho\to\infty} f(\rho) = \lim_{\rho\to\infty} h(\rho) = 1 \end{aligned} \tag{3.21}$$

From (3.20), we have

$$\partial\Psi/\partial x_1 = -x_1/\rho^2 \qquad \text{and} \qquad \partial\Psi/\partial x_2 = x_2/\rho^2 \tag{3.22}$$

Substituting relations (3.18), (3.19), (3.20) and (3.22) into (3.9), we get

$$\begin{aligned} &k_1 = -(v^2 x_2/\rho^2)|q| f^2(\rho)[1 - h(\rho)], \\ &k_2 = (v^2 x_1/\rho^2)|q| f^2(\rho)[1 - h(\rho)], \\ &k_3 = 0, \quad k_4 = 0 \end{aligned} \tag{3.23}$$

Substituting relations (3.18), (3.19) and (3.22) into field equation (3.7), we have

$$f''(\rho) + f'(\rho)/\rho - f(\rho)/\rho^2[1 - h(\rho]^2 + (M_\phi^2/2)[1 - f^2(\rho)]f(\rho) = 0 \tag{3.24}$$

where dash devotes derivatives with respect to ρ. At large distance, in view of (3.21), we may have

$$f(\rho) = 1 - \varepsilon(\rho) \tag{3.25}$$

where $\varepsilon(\rho)$ is infinitesimally small at large distance such that

$$\lim_{\rho\to\infty} \varepsilon(\rho) = 0$$

Then (3.24) may be written as

$$\varepsilon''(\rho) + \varepsilon'(\rho)/\rho - M_\phi^2\varepsilon(\rho) = 0$$

Substituting $r = M_\phi \rho$ into this equation, we get

$$d^2\varepsilon(r)/dr^2 + ({}^1/_r)d\varepsilon(r)/dr - \varepsilon(r) = 0$$

which is modified Bessel's equation of zero order, with its solution given as

$$\varepsilon(r) = AI_0(r) = AI_0(M_\phi\rho) \tag{3.26}$$

where $I_0\ (r)$ is the modified Bessel's function of zero order, defined as

$$I_0(m_\phi\rho) = \sum_{n=0}^{\infty} \frac{(M_\phi\rho/2)^{2n}}{(n!)^2} = J_0(iM_\phi\rho) \tag{3.27}$$

with $J_0\ (x)$ as the ordinary Bessel's function of zero order.

In the similar manner, the field equation (3.8) may be written into the following form by using relations (3.19) and (3.23);

$$h''(\rho) - h'(\rho)/\rho + M_D^2[1 - h(\rho)]f^2(\rho) = 0 \tag{3.28}$$

At large distance we may have

$$h(\rho) = 1 - \zeta(\rho) \tag{3.29}$$

where $\lim_{\rho\to\infty} \zeta(\rho) = 0$.

Then (3.28) reduces to

$$\frac{d^2\zeta(r)}{dr^2} - \frac{d\zeta(r)}{dr} - \zeta(r) = 0 \tag{3.30}$$

where $r = M_D\rho$. Let us substitute $\zeta(r) = r\chi(r)$ is to this equation. Then we have

$$rd^2\chi(r)/dr^2 + d\chi(r)/dr - \chi(r)[1 + 1/r^2] = 0 \tag{3.31}$$

which is modified Bessel's equation of order-one with its solution given by

$$\chi(r) = \zeta(r)/r = BI_1(r) \tag{3.32}$$

where $I_1\,(r)$ is modified Bessel's function of order one defined as

$$I_1(r) = \frac{1}{i} J_1(ir) = \sum_{n=0}^{\infty} \frac{(r/2)^{2n+1}}{n!(n+1)!} \tag{3.33}$$

with $J_1\,(x)$ as the ordinary Bessel's function of first kind of order-one.

Thus we have

$$\zeta(\rho) = B(M_D\rho) I_1(M_D\rho) \tag{3.34}$$

with relations (3.26) and (3.34) the mistakes in the similar relations of Cherinodub et al. [18] stand corrected.

Substituting relations (3.26) and (3.34) into (3.25) and (3.29), we have, at large value of ρ,

$$f(\rho) = 1 - AI_0(M_\phi\rho)$$

and (3.35)

$$h(\rho) = 1 - B(M_D\rho) I_1(M_D\rho)$$

Substituting these results into (3.18) and (3.19), we get the solution of classical field equation (3.7) and (3.8) corresponding to the RCD string with a quark and an anti-quark at its ends. The infinitely separated quark and anti-quark correspond to an axially symmetric solution of the string. For such a string solution with a lowest non-trivial flux the coefficient A in the solution (3.25) is always equal to one while the coefficient B is unity in the Bogomolnyi limit exactly on the border between the type I and type II superconductors [33] where $M_D = M_\phi$ i.e. coherence length and the penetration length coincide with each other. Thus in RCD close to border, we set $B = 1$ besides $A = 1$ and then we have

$$f(\rho) = 1 - I_0(M_\phi\rho) = -\sum_{n=1}^{\infty} \frac{(M_\phi\rho/2)^{2n}}{(n!)^2}$$

and (3.36)

$$h(\rho) = 1 - (M_D\rho) I_1(M_D\rho) = 1 - \frac{(M_D\rho/2)^2}{2} - M_D\rho \sum_{n=1}^{\infty} \frac{(M_D\rho/2)^{2n+1}}{n!(n+1)!}$$

The RCD string is well defined by these solutions. In view of conditions (3.21), the magnetic constituent of the dyonic current, given by (3.23), near the RCD string is zero at the centre of the string (i.e. for $x_1 = x_2 = 0$) and also zero at the points far from the string (where $h(\rho) \to 1$). This current has a maximum at that transverse distance from the string for which the following conditions are satisfied;

$$2(1-h)ff' - f^2h' = 0$$

and (3.37)

$$f^2h'' - 2f'(1-h) > \rho^2 ff''(1-h)$$

4 Superconductivity in Restricted SU(3) Gauge Theory

Let us start with the construction of the restricted chromodynamics in SU(3) limit. The magnetic structure of this theory may be described by two internal Killing vectors which commute with each other and also with the gauge symmetry itself and are normalized to unity according to (2.3). These Killing vectors are a λ_3-like octed $\hat{m}$ and its symmetric product

$$\hat{m}' = \sqrt{3}(m \times \hat{m}) \tag{4.1}$$

which is λ_8-like. The restricted theory (RCD) may be extracted from the full QCD by imposing the extra internal symmetries. Let us restrict the dynamical degrees of freedom of the theory (while keeping the full gauge of freedom intact) by imposing the extra magnetic symmetry which restrict the generalized non-Abelian gauge potential $\vec{\cancel{V}}_\mu$ to satisfy the constraints given by

$$\begin{aligned} D_\mu \hat{m} &= \partial_\mu \hat{m} + i|q|\vec{\cancel{V}}_\mu \times \hat{m} = 0, \\ D_\mu \hat{m}' &= \partial_\mu \hat{m}' + i|q|\vec{\cancel{V}}_\mu \times \hat{m}' = 0 \end{aligned} \tag{4.2}$$

where D_μ is covariant derivative for the gauge group.

Introducing these magnetic structures, we obtain the following form of the generalized restricted potential in the restricted SU(3) gauge theory:

$$\vec{\cancel{V}}_\mu = -iV_\mu^* \hat{m} - iV_\mu^{*\prime} \hat{m}' + (i/|q|)\hat{m} \times \hat{m} + (i/|q|)\hat{m}' \times \partial_\mu \hat{m}' \tag{4.3}$$

where

$$\hat{m}\vec{\cancel{V}}_\mu = -iV_\mu^* \tag{4.4}$$

and

$$\hat{m}' \cdot \vec{\cancel{V}}'_\mu = -iV_\mu^{*\prime}$$

are, respectively the λ_3-like and λ_8-like unrestricted Abelian components of the restricted potential. In the magnetic gauge m and m' become the space-time independent ε_3 and ε_8 respectively, where

$$\hat{\varepsilon}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \hat{\varepsilon}_8 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{4.5}$$

Then the generalized potential of (4.3) may be written as

$$\vec{\cancel{V}}_\mu = (-iV_\mu^* + W_\mu)\hat{\varepsilon}_3 + (-iV_\mu^{*\prime} + W'_\mu)\hat{\varepsilon}_8 \tag{4.6}$$

where W_μ and W'_μ may be identified as the potential of topological dyons in magnetic symmetry of SU(3) gauge theory. These potentials are entirely fixed by $\hat{m}$ and $\hat{m}$ respectively, up to Abelian gauge degrees of freedom, the generalized field strength can, then, be constructed as

$$\begin{aligned} \vec{\cancel{G}}_{\mu\upsilon} &= \vec{G}_{\mu\upsilon} + (i/|q|)[\vec{\cancel{V}}_{\mu} \times \vec{\cancel{V}}_{\upsilon}] \\ &= (-iF_{\mu\upsilon} + H_{\mu\upsilon})\hat{\varepsilon}_3 + (-iF'_{\mu\upsilon} + H'_{\mu\upsilon})\hat{\varepsilon}'_8 \end{aligned} \tag{4.7}$$

where $F_{\mu\upsilon}$ is given by (2.6), $H_{\mu\upsilon}$ is given by (2.9) and

$$\begin{aligned} \vec{G}_{\mu\upsilon} &= \partial_{\mu}\vec{\cancel{V}}_{\upsilon} - \partial_{\upsilon}\vec{\cancel{V}}_{\mu}, \\ F'_{\mu\upsilon} &= \partial_{\mu}V^{*}_{\upsilon} - \partial_{\upsilon}V^{*}_{\mu}, \\ H'_{\mu\upsilon} &= \partial_{\mu}W'_{\mu} - \partial_{\upsilon}W'_{\mu} \end{aligned} \tag{4.8}$$

In this theory the gauge fields are expressible in terms of purely time-like non-singular potentials V^{*}_{μ}, V'^{*}_{μ}, W_{μ} and W'_{μ}. Then in the absence of quarks or any colored object, the RCD Lagrangian of SU(3) theory in magnetic gauge may be written as

$$L = 1/4H_{\mu\upsilon}H^{\mu\upsilon} + 1/4H'_{\mu\upsilon}H'^{\mu\upsilon} + 1/2|D_{\mu}\phi|^2 + 1/2|D'_{\mu}\phi'|^2 - V(\phi^{*}\phi, \phi'^{*}\phi') \tag{4.9}$$

where

$$\begin{aligned} D_{\mu}\phi &= (\partial_{\mu} + i|q|W_{\mu})\phi, \\ D'_{\mu}\phi' &= (\partial_{\mu} + i|q|W'_{\mu})\phi' \end{aligned} \tag{4.10}$$

and the dyonic field operators ϕ and ϕ' corresponds to m and m' respectively. Here $V\ (\phi^{*}\phi, \phi'^{*}\phi')$ is the effective potential introduced to induce the dynamical breaking of the magnetic symmetry. This Lagrangian is a gauge extension of Lagrangian (2.10) and it leads to dyonic condensation, color confinement and the resulting dual superconductivity in SU(3) theory. In the light of the results of Sect. 2, it is not difficult to guess the presence of two scalar modes and two vector modes as the consequence of the presence of two magnetic vectors $\hat{m}$ and m' in SU(3) theory.

5 Discussion

Equations (2.8) and (2.9) show that in the magnetic gauge the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_{μ} by removing all non-essential gauge degrees of freedom in restricted chromo-dynamics (RCD) and hence the topological structure of the theory may be brought into dynamics explicitly. It assures a non-trivial dual structure of the theory of dyons in the magnetic gauge where the gauge fields are expressible in terms of purely time-like non-singular physical potentials V_{μ} and W_{μ}. Thus the topological charge in this theory may be identified as dual object of usual Noetherian charge. Consequently, the restricted theory is expected to lead to better insight of the complicated non-Abelian theory of dyons. The Lagrangian given by (2.10) for RCD in magnetic gauge in the absence of quarks or any colored objects, establishes an analogy between super-conductivity and the dynamical breaking of magnetic symmetry which incorporates the confinement phase in RCD vacuum where the effective potential $V\ (\theta^{*}\theta)$ induces the dyonic condensation of vacuum. This gives rise to dyonic super-current. The electric constituent of this current (i.e. its real part) screens the electric flux and confines the magnetic charges due to usual Meissner effect while its imaginary part (i.e. its magnetic constituent) screens the magnetic flux and confines the color iso-charges as the result of dual Meissner effect. Thus the dynamical breaking of the magnetic symmetry in this theory ultimately induces the generalized Meissner effect with electric constituent as the usual Meissner effect and its magnetic constituent as the dual Meissner effect. It dictates the mechanism for the confinement of the electric and magnetic fluxes associated with dyonic quarks [34] in the present theory.

This dyonic condensation mechanism of confinement implies that long-range physics is dominated by Abelian degrees of freedom (Abelian dominance) as depicted by (2.8) and (2.9) which assure a non-trivial dual structure of the theory of dyons in magnetic gauge, where these objects appear as point like Abelian ones. This idea of Abelian dominance has recently been verified by gauge fixing and Abelian projection [35] and also by constructing semilocal models in Extended Abelian Higgs model (EAH-model) [36, 37]. The same idea has been used, more recently, in connection with the dual Meissner effect in local unitary gauges in SU(2) gluo-dynamics [38] and also with confining ensemble of dyons [39] and dual superconductivity in Yang-Mills theories [40].

In the confinement phase of RCD, the dyons are condensed under the condition (3.2) where the transition from $\langle\phi\rangle_0 = 0$ to $\langle\phi\rangle_0 = v \neq 0$ is of first order, which leads to the vacuum becoming a chromo-magnetic super-conductor in the analogy with Higgs Ginsburg Landau theory of super-conductivity. Dyonically condensed vacuum is characterized by the presence of two massive modes given by (3.3) and (3.4) respectively, where the mass of scalar mode M_ϕ determines how fast the perturbative vacuum around a color source reaches condensation and the mass M_D of vector mode determines the penetration length of the colored flux. With these two mass scales of dual gauge boson and dyonic field, the coherence length ε and the penetration length λ have been constructed by (3.5) in RCD theory. These two lengths coincide at the border between type-I and type-II super-conductors. In general, the ratio of penetration length and coherence length distinguishes superconductors of type-I ($\lambda < \varepsilon$) from type II ($\lambda > \varepsilon$). Equation (3.11) gives the flux penetration depth in the dyonic model of RCD and shows that due to the dynamical breaking of magnetic symmetry, the vacuum acquires the properties similar to those of relativistic super-conductor where the quantum fields generate non-zero expectation values and induces screening currents. This penetration length excludes the generalized field in a manner similar to that in type II super-conductor where the appropriate screening currents are set up by the formation of Cooper's pairs giving rise to Meissner effect of magnetic flux confinement. Thus the generalized color flux is squeezed into flux tubes as a result of generalized Meissner effect caused by the coherence plasma of dyons in RCD vacuum which ultimately forces the quark (color) confinement in RCD. The generation of screening current and the finite range force field responsible for the confinement here are similar to those in the case of real electromagnetic super-conductor (i.e. relativistic superconductor).

Equations (3.16) and (3.17) show that with the suitable choice of the generalized charge space parameter θ, the tubes of generalized confining flux can be made thin which gives rise to a higher degree of confinement of any generalized color flux by dynamically condensed vacuum. These equations demonstrate that the generalized charges lying on the cone of vertical angle $\theta = \pi/4$ in charge space give rise to thin tubes of confined color flux leading to strong confinement of the colored sources in RCD vacuum. On the other hand, the generalized charges lying outside such cone and still participating in the vacuum condensation, immediately after magnetic symmetry breaking, have weak confinement effects. The generalized charge space parameter θ associated with dyons has the remarkable ability to squeeze the color fluxes and to improve the confining properties of RCD vacuum. Thus a perfect confinement can be achieved with pure dyonic states participating in actual dyonic condensation of RCD vacuum as the result of magnetic symmetry breaking in strong coupling limit.

Relations (3.26) and (3.34) removes the mistakes of the similar relations of Chernodub et al. [26]. Substituting relations (3.35) into (3.18) and (3.19), the solutions of classical field equations (3.7) and (3.8), corresponding to the RCD string with a quark and antiquark at its ends, readily follow. The RCD string is well defined by solutions (3.36) where the magnetic constituent of the dyonic current, given by (3.23) near the RCD string, is zero at the centre of the string and also zero at points far away from the string. This current is maximum at the transverse distance for which the conditions (3.37) are satisfied. The numerical value of this distance has been found to be about .2 fm corresponding to SU(2) gluon dynamics [26]. Dyonic density in the absence of string has the contributions from monopole condensate [41, 42] and also from the perturbative fluctuations. According to (3.23) and (3.36) the magnetic constituent of dyonic current at large transverse distance form the string should be controlled by the coherence length and the penetration length where the coherence length could be derived directly [26] from the measurement of dyonic density around a chromodyonic flux spanned between a static quark- anti quark pair. In the maximal Abelian gauge, as used in RCD here, the penetration length and coherence length are almost the same and hence the vacuum is nearly the border between type I and type II dual superconductors. The solutions (3.36) define infinitely long RCD strings which can not be terminated and hence behave like ANO vortices [43] and twisted superconducting semi-local strings [37] with conserved global current flowing through them. Equations (4.2) give the magnetic structure of restricted chromodynamics in SU(3) theory where two internal Killing vectors λ_3-like octed and λ_8-octed given by (4.1) have been introduced keeping in view the facts that any system possessing a SU(3) symmetry suffers with a non-Abelian magnetic instability for the 4–7$^{\text{th}}$ gluons [44] and the 8$^{\text{th}}$ gluon corresponds to the diagonal generator in color space [45]. Equations (4.6) and (4.7) give restricted generalized potential and gauge field strength respectively in the magnetic symmetry of SU(3) gauge theory where the space-time independent octeds ε_3 and ε_8 are given by (4.5). The RCD Lagrangian of SU(3) theory in the absence of quarks or an colored object, is given by (4.9) which appears as gauge extension of SU(2) Lagrangian (2.10). This Lagrangian leads to dyonic condensation, color confinement and the resulting dual superconductivity in SU(3) theory with the presence of two scalar modes and two vectors modes as the consequence of the presence of two magnetic octeds (λ_3-like and λ_8-like) in RCD of SU(3) theory.

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